

Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \frac{1}{3x^2 + 3x - 18} = \frac{1}{(3)(x+3)(x-2)}$

VA $x = -3$ $x = 2$

Holes None

H.A. $y = 0$

x-int None

y-int $y = -1/18$

2) $f(x) = \frac{x-2}{x-4}$

VA $x = 4$

Holes None

H.A. $y = 1$

x-int $x = 2$

y-int $y = 1/2$

3) $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18} = \frac{x(x-3)(x+2)}{-3(x+3)(x-2)}$

VA: $x = -3$ $x = 2$

Holes None

H.A. None

SA $y = -1/3 x$

$-1/3x$
 $-3x^2 - 3x + 18 \overline{) x^3 - x^2 - 6x}$
 $(x^3 + x^2 - 6x)$
 $\underline{\hspace{1cm} 0}$

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12} = \frac{(x+3)(x-2)}{-4(x^2 + 4x + 3)} = \frac{(x+3)(x-2)}{-4(x+3)(x+1)}$

VA: $x = -1$

Hole $x = -3$

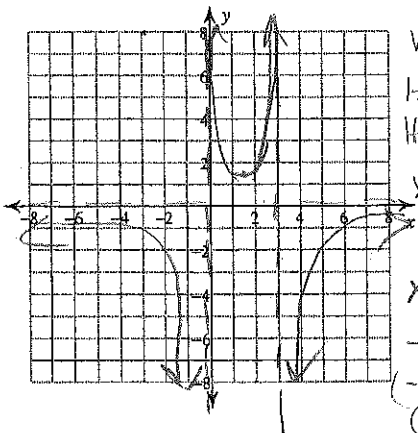
H.A. $y = -1/4$

x-int $x = 2$

y-int $y = 1/2$

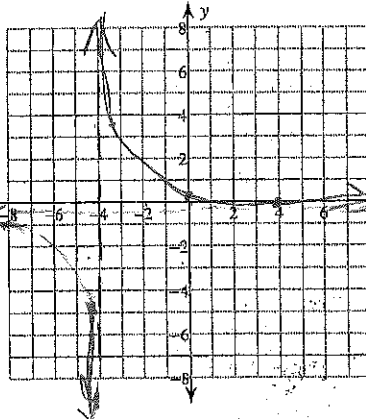
Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

5) $f(x) = -\frac{4}{x^2 - 3x} = -\frac{4}{x(x-3)}$



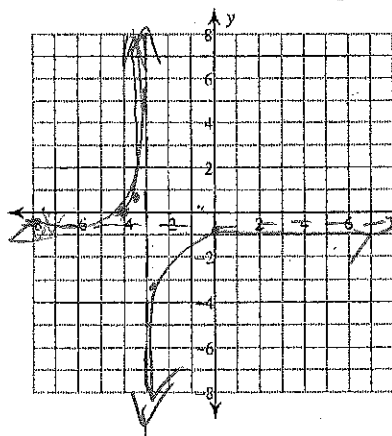
VA $x = 0$ $x = 3$
 Holes: None
 H.A. $y = 0$
 x-int = None
 y-int None
 $x < 0$ or $0 < x < 3$ $x > 3$
 $\frac{-}{(-)} \frac{-}{(+)} \frac{-}{(-)}$

6) $f(x) = \frac{x-4}{-4x-16} = \frac{x-4}{-4(x+4)}$



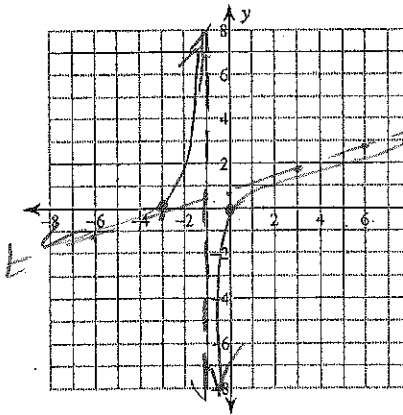
VA $x = -4$
 Holes None
 H.A. $y = -1/4$
 x-int $x = 4$
 y-int $y = 1/4$

7) $f(x) = \frac{x+4}{-2x-6} = \frac{x+4}{-2(x+3)}$



VA $x = -3$
 Holes None
 H.A. $y = -1/2$
 x-int $x = -4$
 y-int $y = -2/3$

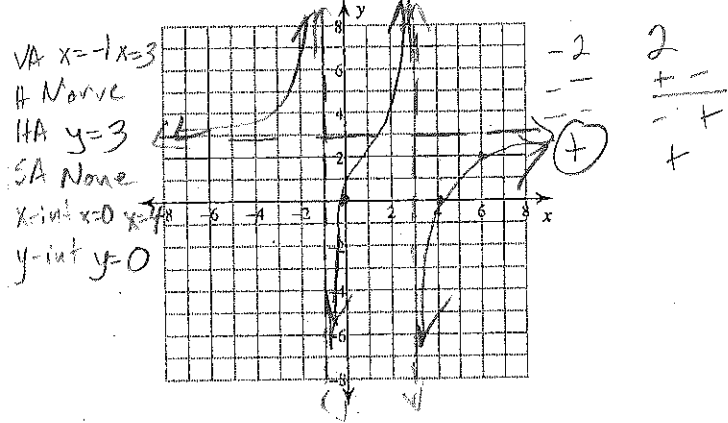
8) $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9} = \frac{x(x^2 - 9)}{3(x^2 - 2x - 3)} = \frac{x(x-3)(x+3)}{3(x-3)(x+1)}$



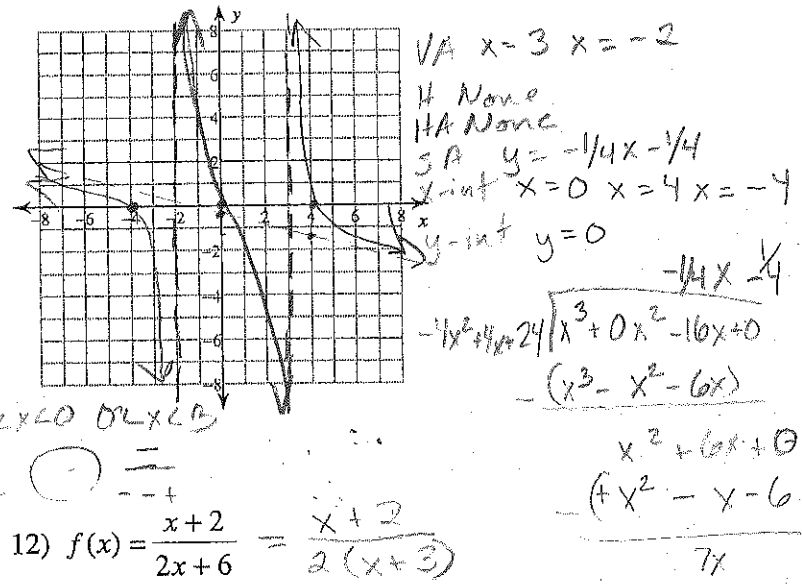
VA $x = -1$
 Holes $x = 3$
 H.A. None
 x-int $x = 0$ $x = -3$
 y-int $y = 0$
 SA, $y = 1/3 x + 2/3$

$\frac{1/3x + 2/3}{3x^2 - 6x - 9} \overline{) x^3 - 10x^2 - 9x}$
 $(x^3 - 2x^2 - 3x)$
 $\underline{\hspace{1cm} 7x^2 - 6x - 0}$
 $\underline{\hspace{1cm} -(2x^2 - 4x - 0)}$
 $\underline{\hspace{1cm} -2x + 0}$

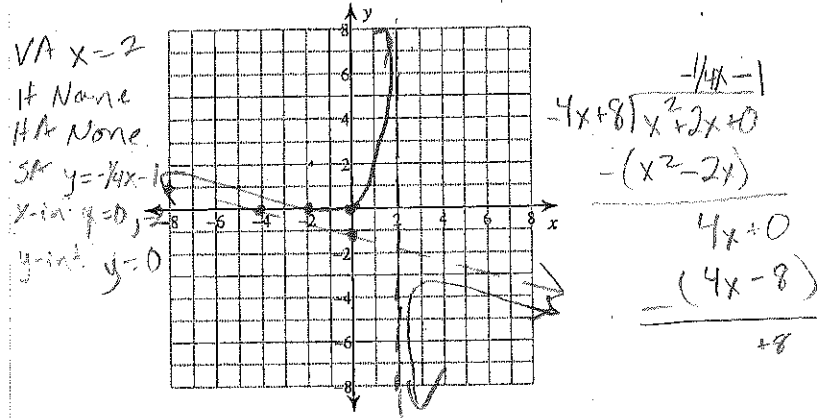
$$9) f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3} = \frac{3x(x-4)}{(x-3)(x+1)}$$



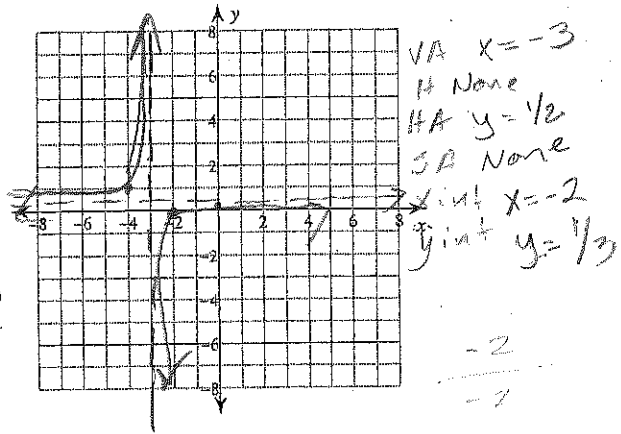
$$10) f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24} = \frac{(x)(x-4)(x+4)}{-4(x-3)(x+2)}$$



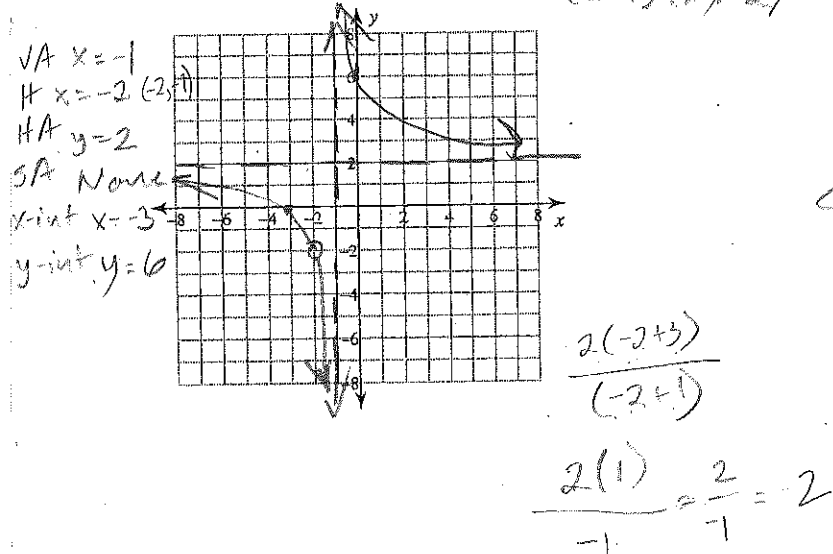
$$11) f(x) = \frac{x^2 + 2x}{-4x + 8} = \frac{x(x+2)}{-4(x-2)}$$



$$12) f(x) = \frac{x+2}{2x+6} = \frac{x+2}{2(x+3)}$$



$$13) f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2} = \frac{2(x+3)(x+2)}{(x+1)(x+2)}$$



$$14) f(x) = \frac{3}{x-2}$$

